CALCULATION OF WEAR FOR A PLAIN BEARING WITH A THIN POROUS ELASTIC SHELL

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The contact problem with wear for a thin ring whose pores are filled with viscous compressible liquid is considered within the scope of the Biot model for a porous elastic material. Over its outer surface the ring is joined with a nondeformable housing, and over part of its inner surface it is in contact with a shaft rotating around its axis. It is assumed that shaft wear is negligibly small compared with that of the bush, inertia effects in the ring may be ignored, and friction force is connected with contact pressure by Coulomb's law. Explicit asymptotic expansions are obtained for the main characteristics of contact interaction (settlement of points of the ring under the shaft, contact angle, contact pressure) valid with short and long periods. Ranges of their link-up are established.

1. We consider a plane (the case of plane strain) contact problem of wear of a thin ring (plain bearing bush) with inner radius R and outer radius R_2. Over the outer contour the ring is joined with a nondeformable bush, and over part of the inner surface it is in contact with a shaft of radius $R_1 = R - \Delta(\Delta R_2^{-1} \ll 1,\ hR_2^{-1} \ll 1)$ rotating around its axis with constant angular velocity ω and transmitting to the bush a force $F(t) = PH(t)$, where $H(t)$ is Heaviside function (see Fig. 1). We assume that shaft wear is negligibly small compared with that of the bush, inertia effects in the ring may be ignored, the force of friction is connected with contact pressure by Coulomb's law, and as shown in [1] the force of friction $\tau_{r\varphi}$, which develops in the contact region with values of friction coefficient $f \leq 0.2$ has little effect on the rule for distribution of contact pressures and the size of the contact angle, and therefore they are not considered in determining radial displacements of the bush.

The rheological properties of the ring material will be described by equations of the Biot model [2] assuming the movement of a viscous (η is viscosity coefficient) compressible liquid in the pores obeys the Darcy filtration rule with a penetration factor k:

$$\mu\left(\nabla^2 u - \frac{u}{r^2}\right) + (\mu + \lambda_c)\frac{\partial e}{\partial r} - \frac{2\mu}{r^2}\frac{\partial v}{\partial \varphi} - \alpha M\frac{\partial \zeta}{\partial r} = 0,$$
$$\mu\left(\nabla^2 v - \frac{v}{r^2}\right) + (\mu + \lambda_c)\frac{1}{r}\frac{\partial e}{\partial \varphi} + \frac{2\mu}{r^2}\frac{\partial u}{\partial \varphi} - \frac{\alpha M}{r}\frac{\partial \zeta}{\partial \varphi} = 0; \tag{1.1}$$

$$\frac{\partial \zeta}{\partial t} = \frac{kM_c}{\eta}\nabla^2\zeta \quad \left(M_c = \frac{M(2\mu + \lambda)}{2\mu + \lambda_c}\right); \tag{1.2}$$

$$p = -\alpha Me + M\zeta, \quad e = \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \varphi}, \tag{1.3}$$

$$\zeta = -\left(\frac{\partial U}{\partial r} + \frac{U}{r} + \frac{1}{r}\frac{\partial V}{\partial \varphi}\right);$$

$$\tau_{ij} = 2\mu e_{ij} + \delta_{ij}(\lambda_c e - \alpha M\zeta), \ \lambda_c = \lambda + \alpha^2 M. \tag{1.4}$$

Here $\mathbf{u} = \{u, v\}$ is the vector of displacements for points of the elastic skeleton; $\mathbf{w} = f(\mathbf{U} - \mathbf{u}) = \{U, V\}$ (U is the vector of displacements of points of the liquid and f is porosity); p is hydrostatic pressure of the liquid in pores; τ_{ij} is stress tensor in a porous material; e_{ij} is the strain tensor in an elastic skeleton (indices i and j cover 1, 2, and here 1 corresponds to r and 2 corresponds to φ); μ, λ, α, and M are elastic coefficients of a porous material whose physical meaning and methods for finding them are given in [3].

The contact conditions between the shaft and the bush as a result of wear of the latter are written in the form [1]

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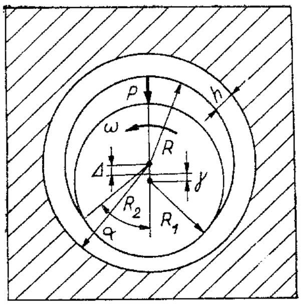

Fig. 1

$$-u(R, \varphi, t) - u_*(R, \varphi, t) = [\Delta + \gamma(t)]\cos\varphi - \Delta \quad (|\varphi| \leqslant \alpha(t)), \tag{1.5}$$

where $u_*(R, \varphi, t)$ is linear wear of the bush in time t over the direction of the radius vector with angular coordinate φ; $\gamma(t)$ is reciprocating movement of the journal under the action of force F(t). In future we shall assume that $0 \leq t < T < \infty$, and T is such that $\gamma(t)$ has the order of displacements in linear elasticity theory.

In order to determine the elastic displacement of points of a ring $u(r, \varphi, t)$ we assume that its inner surface is permeable, and the outer surface is absolutely impermeable. We consider the subsidiary problem of the action of a normal concentrated force $PH(t)$ applied at point $\varphi = 0$, $r = R$ of the bush. Mathematically it is reduced to integrating Eqs. (1.1)-(1.4) with boundary conditions [$\delta(\varphi)$ is Dirac delta-function]

$$\begin{gathered} r = R\colon\ p = 0,\ \tau_{r\varphi} = 0\ (|\varphi| \leqslant \pi),\ \tau_{rr} = -P\delta(\varphi)H(t), \\ r = R_2\colon\ u = v = \partial p/\partial r = 0 \end{gathered} \tag{1.6}$$

and with the initial condition

$$\alpha e = \zeta - M^{-1}p \ (t = 0). \tag{1.7}$$

In order to solve (1.1)-(1.4), (1.6), (1.7) we introduce two unknown functions $E(r, \varphi, t)$ and $S(r, \varphi, t)$ connected with displacement vector components in an elastic skeleton $\{u, v\}$ and pressure p by the expressions

$$\begin{gathered} u = \frac{\partial E}{\partial r} + \sin\varphi\left(r\frac{\partial S}{\partial r} - S\right), \\ v = \frac{1}{r}\frac{\partial E}{\partial \varphi} + \sin\varphi\frac{\partial S}{\partial \varphi} - S\cos\varphi, \\ p\alpha = (2\mu + \lambda)\nabla^2 E + 2\mu\left(\sin\varphi\frac{\partial S}{\partial r} + \frac{\cos\varphi}{r}\frac{\partial S}{\partial \varphi}\right), \quad e = \nabla^2 E. \end{gathered} \tag{1.8}$$

By substituting (1.8) in (1.1)-(1.3) we arrive at the conclusion that the original equations are converted to an identity if functions S and E satisfy the equations

$$\nabla^2 S = 0,\ \frac{\partial}{\partial t}\nabla^2 E = c\nabla^4 E \quad \left(c = \frac{kM_c}{\eta}\right). \tag{1.9}$$

By inserting (1.8) in (1.4) we obtain

$$\begin{gathered} \tau_{rr} = 2\mu\left[-\frac{1}{r}\left(\frac{\partial E}{\partial r} + \frac{1}{r}\frac{\partial^2 E}{\partial \varphi^2}\right) + \sin\varphi\left(r\frac{\partial^2 S}{\partial r^2} - \frac{\partial S}{\partial r}\right) - \frac{\cos\varphi}{r}\frac{\partial S}{\partial \varphi}\right], \\ \tau_{r\varphi} = \frac{2\mu}{r}\left[\frac{\partial}{\partial \varphi}\left(\frac{\partial E}{\partial r} - \frac{E}{r}\right) + r\sin\varphi\frac{\partial}{\partial \varphi}\left(\frac{\partial S}{\partial r} - \frac{S}{r}\right)\right]. \end{gathered} \tag{1.10}$$

Now by drawing attention to Eqs. (1.8) and (1.10) we transform (1.6) and (1.7) to the form

$$\begin{gathered} r = R\colon\ (2\mu + \lambda)\nabla^2 E + 2\mu\left(\sin\varphi\frac{\partial S}{\partial r} + \frac{\cos\varphi}{r}\frac{\partial S}{\partial \varphi}\right) = 0, \\ \frac{\partial}{\partial \varphi}\left(\frac{\partial E}{\partial r} - \frac{E}{r}\right) + r\sin\varphi\frac{\partial}{\partial \varphi}\left(\frac{\partial S}{\partial r} - \frac{S}{r}\right) = 0, \end{gathered}$$

$$-\frac{1}{r}\left(\frac{\partial E}{\partial r}+\frac{1}{r}\frac{\partial^2 E}{\partial \varphi^2}\right)+\sin\varphi\left(r\frac{\partial^2 S}{\partial r^2}-\frac{\partial S}{\partial r}\right)-\frac{\cos\varphi}{r}\frac{\partial S}{\partial \varphi}=-\frac{P}{2\mu}\delta(\varphi)H(t)$$

$$r=R_2:\ \frac{\partial E}{\partial r}+\sin\varphi\left(r\frac{\partial S}{\partial r}-S\right)=0,\quad \frac{1}{r}\frac{\partial E}{\partial \varphi}+\sin\varphi\frac{\partial S}{\partial \varphi}-S\cos\varphi=0; \tag{1.11}$$

$$(2\mu+\lambda)\frac{\partial}{\partial r}\nabla^2 E+2\mu\left(\sin\varphi\frac{\partial^2 S}{\partial r^2}-\frac{\cos\varphi}{r^2}\frac{\partial S}{\partial \varphi}+\frac{\cos\varphi}{r}\frac{\partial^2 S}{\partial r\,\partial \varphi}\right)=0,$$

$$\nabla^2 E=-\frac{2\mu}{2\mu+\lambda}\left(\sin\varphi\frac{\partial S}{\partial r}+\frac{\cos\varphi}{r}\frac{\partial S}{\partial \varphi}\right)+\frac{\alpha p}{2\mu+\lambda}\quad (t=0). \tag{1.12}$$

We apply to both parts of (1.9), (1.11), and (1.12) the Laplace–Carson integral transformation with respect to time

$$E=\frac{1}{2\pi i}\int\limits_L \frac{E^L}{s}e^{st}\,ds,\quad S=\frac{1}{2\pi i}\int\limits_L \frac{S^L}{s}e^{st}\,ds \tag{1.13}$$

and we shall find functions $E^L(r,\ \varphi,\ s)$ and $S^L(r,\ \varphi,\ s)$ in representation (1.13) in the form

$$E^L=\frac{E_0^L}{2}+\sum_{n=1}^{\infty}E_n^L\cos n\varphi,\quad S^L=\sum_{n=1}^{\infty}S_n^L\sin n\varphi,$$

$$E_n^L(r,\ s)=\frac{1}{\pi}\int\limits_{-\pi}^{\pi}E^L\cos n\varphi\,d\varphi,\quad S_n^L(r,\ s)=\frac{1}{\pi}\int\limits_{-\pi}^{\pi}S^L\sin n\varphi d\varphi. \tag{1.14}$$

By inserting Eqs. (1.13) and (1.14) in Eqs. (1.9) and solving the simple differential equations obtained, we find that

$$E_n^L=A_n r^n+B_n r^{-n}+C_n r^{\gamma_n}+D_n r^{-\gamma_n},$$
$$S_n^L=E_n r^n+F_n r^{-n},\quad \gamma_n^2=n^2+s/c.$$

By satisfying the selection of functions $A_n(s)$, $B_n(s)$, $C_n(s)$, $D_n(s)$, $E_n(s)$, and $F_n(s)$ with boundary conditions (1.11) written in terms of the Laplace–Carson transformation and limiting ourselves to retaining in the solution terms of the order of $O(\Lambda)$ ($\Lambda = hR_2^{-1} \ll 1$), which specify deformation of an elastic skeleton considering that in a thin layer with $t = 0$, $p = f^{-1}P\delta(\varphi)$, we have

$$u^L(R,\ \varphi,\ s)=-\frac{\varepsilon h}{\mu}\left(c_1+c_2\frac{\operatorname{th}\sqrt{ms}}{\sqrt{ms}}\right)P\delta(\varphi),$$

$$m=\frac{h^2}{c},\quad c_1=1-c_2,\quad c_2=\frac{\alpha}{f}<1,\quad \varepsilon=\frac{1-2\nu}{2(1-\nu)} \tag{1.15}$$

(ν is Poisson's ratio for the skeleton material).

Moving to the case of a normal load $q(\varphi)$ distributed in section $|\varphi| \le \alpha$, from equality (1.15) we find that

$$u^L(R,\ \varphi,\ s)=-\frac{\varepsilon h}{\mu}\left(c_1+c_2\frac{\operatorname{th}\sqrt{ms}}{\sqrt{ms}}\right)q(\varphi)\quad (|\varphi|\leqslant\alpha). \tag{1.16}$$

Whence it follows that a relatively thin porous elastic annulas layer operates in compression similar to a Fuss–Winkler viscoelastic base with an operator coefficient of the bed whose form may be determined by taking with respect to both parts of (1.16) an inverse Laplace–Carson transformation [4]. As a result of this

$$u(R,\ \varphi,\ t)=-\frac{\varepsilon h}{\mu}q(\varphi)\left[c_1+\frac{c_2}{m}\int\limits_0^t\theta_2\left(0,\ \frac{t-\tau}{m}\right)d\tau\right],$$

$$\theta_2(x,\ y)=2\sum_{n=0}^{\infty}\exp\left[-\pi^2\left(n+\frac{1}{2}\right)^2 y\right]\cos\pi(2n+1)x \tag{1.17}$$

[$\theta_2(x, y)$ is the theta-function].

The solution for an instantaneous load $\tau_{rr} = -q(\varphi)\delta(t)$, applied the section $|\varphi| \le \alpha$ of the inner ring surface is found by differentiating (1.17) with respect to t:

$$\dot{u}(R, \varphi, t) = -\frac{\varepsilon h}{\mu} q(\varphi)\left[c_1\delta(t) + \frac{c_2}{m}\theta_2\left(0, \frac{t}{m}\right)\right]. \tag{1.18}$$

If in expressions (1.18) $tm^{-1} \to \infty$, then

$$\dot{u}(R, \varphi, t) = -\frac{\varepsilon h}{\mu} q(\varphi)\left[c_1\delta(t) + \frac{2c_2}{m}\exp\left(-\frac{\pi^2 t}{4m}\right)\right]. \tag{1.19}$$

With $tm^{-1} \to 0$ taking account of the fact that

$$u^L(R, \varphi, s) \sim -\frac{\varepsilon h}{\mu}\left(c_1 + \frac{c_2}{\sqrt{ms}}\right)q(\varphi),$$

we obtain [4]

$$\dot{u}(R, \varphi, t) = -\frac{\varepsilon h}{\mu} q(\varphi)\left[c_1\delta(t) + \frac{c_2}{\sqrt{\pi m t}}\right]. \tag{1.20}$$

We assume that bush wear has an abrasive nature [5]. Then its wear rate is proportional to the work of friction force and in the case of $\tau_{rr} = -q(\varphi)$ it takes the form

$$\dot{u}_*(R, \varphi, t) = -lR_2 q(\varphi)\mu^{-1}. \tag{1.21}$$

Here l is a constant which characterizes ring material wear resistance, conditions for operation of the shaft-bush pair, and depends on the combination of wearing surfaces.

2. Knowing functions $u(R, \varphi, t)$ and $\dot{u}_*(R, \varphi, t)$ (1.18)-(1.21), we study the contact problem stated in Part 1. Since with the passage of time the contact area of the journal with the bearing $2\alpha(t)$ increases uniformly, then there exists a function $t = \beta(\alpha)$ inverse to $\alpha = \alpha(t)$, and its uniqueness makes it possible to use $\alpha(t)$ as a provisional parameter. Using contact condition (1.5) and denoting

$$t^* = tm^{-1},\quad \Delta^* = \Delta R_2^{-1},\quad b = c_2 c_1^{-1},\ \gamma^*(t^*) = \gamma[\alpha(t)]R_2^{-1},\quad q^*(\varphi, t^*) =$$
$$= q[\varphi, \alpha(t)]\mu^{-1},\quad \varepsilon\Lambda c_1 = a,\quad l^* = lm(ab)^{-1},\quad N = P(\mu R_2)^{-1}$$

(the asterisk is omitted below), we obtain an integral equation for the problem of relatively unknown contact pressure $q(\varphi, t)$ in the form

$$a\left[q(\varphi, t) + b\int_0^t q(\varphi, \tau)k(t-\tau)\,d\tau\right] = [\Delta + \gamma(t)]\cos\varphi - \Delta \quad (0 \leqslant \varphi \leqslant \alpha(t),$$
$$0 \leqslant t \leqslant T < \infty), \tag{2.1}$$

where the kernel $k(t - \tau)$ gives one of the equations

$$k(t) - l = \theta_2(0, t),\ 2\exp(-\pi^2 t/4),\ (\pi t)^{-1/2} \tag{2.2}$$

for versions (1.18)-(1.20), respectively. To Eqs. (2.1) and (2.2) it is necessary to add the quasistatic condition

$$N = 2\int_0^{\alpha(t)} q(\varphi, t)\cos\varphi\, d\varphi, \tag{2.3}$$

and also the equality

$$q(\varphi, t) = 0 \quad (\varphi \geqslant \alpha), \tag{2.4}$$

which serves for determining the unknown contact zone of the shaft and the bush.

It is noted that relationship (2.4) makes it possible to write integral Eq. (2.1) in the form of the system

TABLE 1

t	$\alpha(t)$			t	$\alpha(t)$		
0	0,998	0,998	0,998	0,8	1,158	1,171	1,135
0,05	1,053	1,053	1,021	0,9	1,162	1,179	1,139
0,1	1,073	1,073	1,039	1	1,165	1,186	1,143
0,2	1,098	1,099	1,067	2	1,182	1,234	1,163
0,3	1,116	1,117	1,088	3	1,193	1,265	1,174
0,4	1,129	1,131	1,102	4	1,202	1,288	1,184
0,5	1,139	1,143	1,113	5	1,211	1,306	1,194
0,6	1,147	1,154	1,122	10	1,249	1,361	1,233
0,7	1,153	1,163	1,129	∞	1,571	1,571	1,571

$$a\left[q(\varphi, t) + b\int_{\psi(\varphi)}^{t} q(\varphi, \tau)\, k(t-\tau)\, d\tau\right] = [\Delta + \gamma(t)]\cos\varphi - \Delta$$
$$(\psi(\varphi) \leqslant t \leqslant T < \infty),$$
$$\psi(\varphi) = \begin{cases} 0 & (0 \leqslant \varphi \leqslant \alpha_0), \\ \beta(\varphi) & (\alpha_0 < \varphi \leqslant \alpha), \end{cases} \quad \alpha_0 = \alpha(0), \tag{2.5}$$

for which an algorithm given in [6, 7] is used for the solution.

From (2.5) by means of (2.4) we find that

$$\gamma(t) = \Delta[1 - \cos\alpha(t)]\cos^{-1}\alpha(t). \tag{2.6}$$

We multiply both parts of (2.1) by $\cos\varphi$ and we integrate within the limits from 0 to $\alpha(t)$. Drawing attention to Eqs. (2.2), (2.3), and (2.6) and changing the order of integration, which is correct [8] in the case of a monotonically increasing contact region, we have

$$\frac{\Delta}{abN}\,\frac{\alpha - (1/2)\sin 2\alpha}{\cos\alpha} - \frac{1}{b} - lt = \begin{cases} 1 - \dfrac{2}{\pi^2}\displaystyle\sum_{n=0}^{\infty}\frac{e^{-\pi^2(n+1/2)^2 t}}{(n+1/2)^2}, \\ 2\sqrt{\dfrac{t}{\pi}} \quad (t \to 0), \\ \dfrac{8}{\pi^2}\left(1 - e^{-\pi^2 t/4}\right) \quad (t \to \infty). \end{cases} \tag{2.7}$$

The last expressions are transcendental equations for determining the contact angle of the journal and the bearing, respectively, with $0 \le t \le T < \infty$, $T_1 \le t \le T$, and $0 \le t \le T_0$. In particular, it follows from (2.7) that with quite a long period $\alpha(t) \to \pi/2$.

A solution for set of integral Eqs. (2.5) with $T_1 \le t \le T$ may be obtained by the method in [6, 7]:

$$q(\varphi, t) = \frac{\Delta}{a}\left[\frac{\cos\varphi - \cos\alpha(t)}{\cos\alpha(t)} + \frac{1}{p_1 - p_2}\sum_{n=1}^{2}(-1)^n p_n \times \right.$$
$$\left. \times \left(p_n - \frac{\pi^2}{4}\right)\int_0^t \frac{\cos\varphi - \cos\alpha(\tau)}{\cos\alpha(\tau)} e^{-p_n(t-\tau)}\, d\tau\right] \quad (0 \leqslant \varphi \leqslant \alpha_0),$$
$$q(\varphi, t) = \frac{\Delta}{a(p_1 - p_2)}\sum_{n=1}^{2}(-1)^n p_n\left(p_n - \frac{\pi^2}{4}\right) \times$$
$$\times \int_{\beta(\varphi)}^{t}\frac{\cos\varphi - \cos\alpha(\tau)}{\cos\alpha(\tau)} e^{-p_n(t-\tau)} d\tau \quad (\alpha_0 < \varphi \leqslant \alpha(t)),$$
$$p_{1,2} = \frac{1}{2}\left[l + 2b + \frac{\pi^2}{4} \mp \sqrt{\left(l + 2b + \frac{\pi^2}{4}\right)^2 - l\pi^2}\right], \tag{2.8}$$

and function $\beta(\varphi)$ in Eqs. (2.8) satisfies the equation

$$\Phi-\frac{1}{b}-l\beta=\frac{8}{\pi^2}\left(1-e^{-\pi^2\beta/4}\right),\quad \Phi(\varphi)=\frac{\Delta}{abN}\frac{\varphi-(1/2)\sin 2\varphi}{\cos\varphi}\quad(\alpha_0<\varphi\leqslant\alpha). \tag{2.9}$$

With $0 \le t \le T_0$ the solution of system (2.5) may be constructed by means of an integral Laplace–Carson transformation with respect to time. Omitting the mathematical computation we write

$$q(\varphi,t)=\frac{\Delta}{a}\left\{\frac{\cos\varphi-\cos\alpha(t)}{\cos\alpha(t)}-\frac{1}{s_1-s_2}\sum_{n=1}^{2}(-1)^n\times\right.$$

$$\left.\times s_n^2\int_0^t\frac{\cos\varphi-\cos\alpha(\tau)}{\cos\alpha(\tau)}\left[s_ne^{s_n^2(t-\tau)}\operatorname{erfc}\left(s_n\sqrt{t-\tau}\right)-\frac{1}{\sqrt{\pi(t-\tau)}}\right]d\tau\right\}\quad(0\leqslant\varphi\leqslant\alpha_0),$$

$$q(\varphi,t)=-\frac{\Delta}{a(s_1-s_2)}\sum_{n=1}^{2}(-1)^n s_n^2\int_{\beta(\varphi)}^{t}\frac{\cos\varphi-\cos\alpha(\tau)}{\cos\alpha(\tau)}\times$$

$$\times\left[s_ne^{s_n^2(t-\tau)}\operatorname{erfc}\left(s_n\sqrt{t-\tau}\right)-\frac{1}{\sqrt{\pi(t-\tau)}}\right]d\tau\quad(\alpha_0<\varphi\leqslant\alpha(t)),$$

$$s_{1,2}=\frac{b}{2}\mp\sqrt{\frac{b^2}{4}-lb},\quad \beta(\varphi)=\frac{\pi(\Phi-1/b)^2}{[1+\sqrt{1+\pi l(\Phi-1/b)}]^2}. \tag{2.10}$$

Here $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$; $\operatorname{erf}(x)$ is probability integral.

Thus, Eqs. (2.6)-(2.10) give a solution of the stated problem for wear of a plain bearing with a porous lining with $T_1 \le t \le T$ and $0 \le t \le T_0$. We clarify the question of whether the equations for short and long periods obtained link up together, i.e., whether the condition $T_1 \le T_0$ is fulfilled?

For example, we assume in relationships (2.7) that $\Delta(aN)^{-1} = 1$, $b = 1$, $l = 0.1$. Values of $\alpha(t)$ found from Eqs. (2.7) ($t \to 0$ is the second column, $t \to \infty$ is the third column) with different values of $0 \le t < \infty$ are given in Table 1. It can be seen that the asymptotic for a short period operates almost to $t = T_0 = 1$ (the error in this solution compared with the accurate solution given in the first column with $t = T_0$ does not exceed 1.8%). At the same time, the asymptotic solution with $t \to \infty$ may be used when $t = T_1 \ge 0.8$ (the maximum error of the results is not more than 2%).

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